Kittel TP

3.1

(a). $Z = \sum_s \exp[-\varepsilon_s/\tau]$

$= 1 + \exp[-\varepsilon/\tau]$.

$$F = -\tau \ln Z = \boxed{-\tau \ln\{1+\exp[-\varepsilon/\tau]\}}$$

(b). $$\sigma = -\left(\frac{\partial F}{\partial \tau}\right)_V = \ln\{1+\exp[-\varepsilon/\tau]\} + \tau\{1+\exp[-\varepsilon/\tau]\}^{-1} \exp[-\varepsilon/\tau] \times \left(-\frac{\varepsilon}{\tau^2}\right)$$

$$= -\frac{\varepsilon}{\tau^2}(\exp[-\varepsilon/\tau])\,\tau\{1+\exp[-\varepsilon/\tau]\}^{-1} + \ln\{1+\exp[-\varepsilon/\tau]\}$$

$$= \boxed{-\frac{\varepsilon}{\tau}\frac{\exp[-\varepsilon/\tau]}{1+\exp[-\varepsilon/\tau]} + \ln\{1+\exp[-\varepsilon/\tau]\}}$$

From the differential relation $U = F + \tau\sigma$, we have

$$U = \tau\ln\{1+\exp[-\varepsilon/\tau]\} - \frac{\varepsilon\exp[-\varepsilon/\tau]}{1+\exp[-\varepsilon/\tau]} - \tau\ln\{1+\exp[-\varepsilon/\tau]\}$$

$$= \boxed{-\frac{\varepsilon\exp[-\varepsilon/\tau]}{1+\exp[-\varepsilon/\tau]}}$$

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